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# Quantum relaxation of the Higgs mass

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## Abstract

I put forward a qualitatively new dynamical mechanism for solving the electroweak hierarchy problem that does not require new physics at the electroweak. I argue that the infrared fluctuations of the gravitational field may provide a partial screening of the Higgs mass, similar to the infrared screening of the electric charge in quantum electrodynamics.

The discovery of the 125 GeV Higgs boson with properties consistent with an elementary scalar, together with non-observation of any sign of new physics at the LHC so far, makes a search for qualitatively novel realisations of the naturalness paradigm paramount. One interesting theoretical possibility has been recently proposed in [1]. According to this proposal the Higgs mass is driven to a value much smaller than the ultraviolet cut-off [being as large as  $\sim 10^8$  GeV] by a peculiar Higgs-axion dynamics in the early inflationary universe.

I would like to propose another unorthodox solution to the mass hierarchy problem, which is inspired by Polyakov's idea on the infrared screening of the cosmological constant [2] (see also [3]). The purpose of this brief note is to demonstrate the key aspect of the mechanism, without neither going into much phenomenological details, nor attempting at ultraviolet completion to establish a fully consistent theory. To this end, I consider an effective theory of the electroweak Higgs doublet  $\Phi$  coupled to gravity valid at some high-energy scale  $\mu_{UV}$ . The effective action reads:

$$S_{\mu_{UV}} = \int d^4x \sqrt{-g} \left[ \frac{R(g)}{16\pi G_N} + \partial_\mu \Phi^\dagger \partial^\mu \Phi - V(\Phi^\dagger \Phi) + \dots \right], \quad (1)$$

where,

$$V(\Phi^\dagger\Phi) = \Lambda + M_\Phi^2\Phi^\dagger\Phi + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2, \quad (2)$$

is the Higgs potential with a cosmological constant  $\Lambda$  and the ellipsis meant to contain all the remaining terms, including high-dimensional operators which are allowed by symmetries of the theory. These terms are irrelevant for the sake of the argument and I ignore them. All the tree-level parameters in the above action ( $\Lambda, M_\Phi, \lambda, \dots$ ) are defined at the cut-off scale  $\mu_{UV}$ . This implicitly assumes that they have been renormalised within a more fundamental theory valid beyond  $\mu_{UV}$ . Therefore, one naturally expects that  $|M_\Phi| \sim |\Lambda|^{1/4} \sim \mu_{UV}$  at the cut-off scale.<sup>1</sup> In other words, the model is not tuned.

The theory (1) is nonrenormalisable and this complicates the discussion within the full general relativistic framework. Instead, following [2], I restrict the full metric  $g_{\mu\nu}$  to be a conformally flat metric:

$$g_{\mu\nu} = \phi^2 \eta_{\mu\nu}, \quad (3)$$

where  $\eta_{\mu\nu}$  is just the flat Minkowski metric. For conformally flat spacetime (3) one computes:

$$R(g) = -6\phi^{-3}\square\phi, \quad \sqrt{-g} = \phi^4, \quad (4)$$

where  $\square$  is the flat spacetime D'Alembertian operator. It is convenient to introduce a new set of fields obtained from the old one by the following field redefinitions:

$$S = \sqrt{6}M_P\phi, \quad H = \phi\Phi, \quad (5)$$

where  $M_P = 1/\sqrt{8\pi G_N} \approx 2.4 \cdot 10^{18}$  GeV is the reduced Planck mass. The action (1) then can be rewritten as:

$$S_{\mu_{UV}} = \int d^4x \left[ \frac{1}{2}\partial_\mu S \partial^\mu S + \mathcal{D}_\mu H^\dagger \mathcal{D}^\mu H - \frac{\lambda}{2}(H^\dagger H)^2 - \frac{\lambda_M}{2}S^2 H^\dagger H - \frac{\lambda_\Lambda}{4}S^4 + \dots \right], \quad (6)$$

where  $\mathcal{D}_\mu = \partial_\mu - \frac{1}{S}\partial_\mu S$ . In Eq. (6), and in what follows, the Lorentz indices are contracted using the Minkowski metric. Note that the Higgs mass term and the cosmological constant turn into marginal operators and the corresponding parameters become dimensionless:

$$\lambda_M(\mu_{UV}) = \frac{M_\Phi^2}{3M_P^2}, \quad (7)$$

$$\lambda_\Lambda(\mu_{UV}) = \frac{\Lambda}{9M_P^4}. \quad (8)$$

Hence, the action (6) exhibits a manifest classical scale invariance,  $x^\mu \rightarrow \xi x^\mu$ ,  $S \rightarrow \xi^{-1}S$ ,  $H \rightarrow \xi^{-1}H$ , which is softly broken upon quantisation by the logarithmic dependence of the dimensionless couplings on the renormalisation scale  $\mu$  ( $\mu < \mu_{UV}$ ). In

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<sup>1</sup>This is not necessarily true. If certain symmetries, such as supersymmetry or scale invariance, are enforced upon the effective low-energy theory from its UV completion, the effective Higgs mass parameter  $M_\Phi$  can be smaller than the ultraviolet cut-off [4]. This is the standard paradigm.

particular, the renormalisation group (RG) running of (7) and (8) are determined, respectively, by the following one-loop RG  $\beta$ -functions:

$$\beta_{\lambda_M} = \frac{\lambda_M}{16\pi^2} (\lambda_M + \lambda_\Lambda + 3\lambda) , \quad (9)$$

$$\beta_{\lambda_\Lambda} = \frac{1}{64\pi^2} (\lambda_M^2 + 2.5\lambda_\Lambda^2) , \quad (10)$$

where we have omitted contributions from Higgs-Yukawa and gauge interactions for simplicity. Note, that (9) exhibits fixed-point of the RG flow at  $\lambda_M = 0$ , which corresponds to the natural decoupling limit between gravity ( $S$ ) and the Higgs sectors [5]. For  $\beta_{\lambda_M} > 0$  this is an infrared fixed-point. Also, according to Eq. (10),  $\lambda_M = 0$  fixed point triggers  $\lambda_\Lambda \approx 0$  (quasi) fixed-point. This RG behaviour is at the heart of the infrared quantum relaxation of the cosmological constant [2] and the electroweak scale simultaneously. More specifically, assuming  $\lambda_M \gg \lambda_\Lambda, \lambda$ , we can easily solve the RG equation for  $\lambda_M$ . Setting the infrared scale at the Higgs mass,  $\mu = M_H$ , we find:

$$\lambda_M(M_H) \approx \frac{16\pi^2}{\ln\left(\frac{M_H}{\mu_{UV}}\right)} . \quad (11)$$

To obtain the desired Planck/weak scale hierarchy one must have  $\lambda_M(M_H) \sim 10^{-32}$ , which for the above perturbative result imply  $\mu_{UV} \gg M_P$ . It could be though that in these regime the leading log perturbative calculations must be improved by employing non-perturbative techniques [3]. Also, although I have no proof, but it seems unlikely that the hierarchy generated in (11) can be undone through the fluctuations of the remaining unimodular part of the metric tensor [2].

As an alternative model, one can consider a possibility for gravity to propagate in extra spatial dimensions. In that case the gravitational fluctuations will induce faster, power-law running of  $\lambda_M$  (as oppose to logarithmic running in 4 dimensions) and hence the desired hierarchy will be generated for smaller values of the ultraviolet cut-off [6].

To conclude, I demonstrate (at least qualitatively) that the electroweak mass hierarchy may originate due to the infrared quantum fluctuations of the gravitational field which triggers quantum relaxation of the Higgs mass and perhaps the cosmological constant as well. It would be interesting to study this proposal in more quantitative manner.

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## References

- [1] P. W. Graham, D. E. Kaplan and S. Rajendran, arXiv:1504.07551 [hep-ph].
- [2] A. M. Polyakov, Sov. Phys. Usp. **25**, 187 (1982) [Usp. Fiz. Nauk **136**, 538 (1982)]; Phys. Atom. Nucl. **64**, 540 (2001) [Yad. Fiz. **64**, 594 (2001)] [Int. J. Mod. Phys. A **16**, 4511 (2001)] [hep-th/0006132].
- [3] R. Jackiw, C. Nunez and S.-Y. Pi, Phys. Lett. A **347**, 47 (2005) [hep-th/0502215].
- [4] A. Kobakhidze and K. L. McDonald, JHEP **1407**, 155 (2014) [arXiv:1404.5823 [hep-ph]].
- [5] R. Foot, A. Kobakhidze, K. L. McDonald and R. R. Volkas, Phys. Rev. D **89**, no. 11, 115018 (2014) [arXiv:1310.0223 [hep-ph]].
- [6] M. Chaichian and A. B. Kobakhidze, Phys. Lett. B **478**, 299 (2000) [hep-th/9912193].